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Heat and mass transfer to a turbulent falling film—II

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INTRODUCTION

This is a continuation of a previous note [1] on the heat and mass transfer for the turbulent film flow of a binary solution down a wall, with the free surface in contact with the vapor phase of the solvent. In the previous note, the assumption that the transfer coefficients at the surface and at the wall are invariant in the direction of the film flow enabled an algebraic solution for the distributions of the mean concentration and mean temperature for the cases of a constant wall temperature or a constant heat flux at the wall. These results were compared with the numerical solution by Grossman and Heath [2], using for the algebraic solution the asymptotic values of the transfer coefficients for long flow lengths given by these authors [3] to partially correct errors in the coefficients that were given in ref. [2]. The algebraic solution agreed with the numerical solution for the Reynolds number of 10^4, but did not do so for the higher Reynolds number of 10^5, for which the numerical results were presented in ref. [2]. The need for additional results caused a resumption of a numerical solution like that of ref. [4] for heat transfer alone, which had failed when used for the combined heat and mass transfer of the present problem. Changes in longitudinal and lateral incrementing, and the reformulation of the interface boundary condition did not improve the energy and mass balances made, for each longitudinal increment, to test the calculation. Resolution was finally achieved by using double precision; the source of the arithmetic difficulty was not found and the disadvantage of the double precision requirement was accepted for the limited number of results that were obtained.

RESULTS

The turbulence model was that of Van Driest, modified by the Mills and Chung eddy diffusivity specifications for the region of the interface; this is Model 1 of ref. [4] except that the Habib and Na diffusivity modification was used as in Model 3 of that reference. Model 3 should have been used entirely but the results of the modified model are acceptable for the comparison of the algebraic and numerical results. The numerical calculations were made for the condition of a constant wall temperature equal to the initial solution temperature, a Lewis number of 200 and an interface flux ratio parameter, λ, equal to 0.01. The Reynolds numbers were 5000, 10^4 and 10^5 while the Prandtl numbers were 1 and 10. The interface diffusivity model requires other parameters, and these were specified for water at a temperature of 25°C. For a Reynolds number of 10^4, this gives the same diffusivity distribution as does the model of ref. [2], that model only being applicable for water at a temperature like this, as noted in ref. [1]. The inconsistency of this with the Prandtl number chosen for the calculations makes the results illustrative only; for any actual situation the parameters must be consistent with the particular operating condition.

Table 1 gives the values of the mean concentration, $\bar{\gamma}$, the mean temperature, $\bar{\theta}$, and the interface temperature, θ_i, as obtained from the algebraic solution given in ref. [1] and also those obtained from the numerical calculation, which are subscripted with N. Table 2 contains the asymptotic values of the Nusselt number, Nu_i, the Sherwood number, Sh_i, for the interface and the Nusselt number, Nu_0, for the wall as given by the numerical results. Associated with each of these is the distance, ξ_1, at which the interface numbers have decreased to 1.05 times their asymptotic values, and at which the wall Nusselt number has increased to 0.95 times its asymptotic value. Table 2 also contains the distance, ξ_2, at which the mean concentration attains the value of 0.90. Since $x/\Delta = 0.25 Re\,\xi$, the film lengths, x, are obtainable when the film thickness is specified. As an example, the table contains the values of the film length, x_2, for water at 25°C flowing down a vertical wall. These are very large and they indicate that concentrations like $\bar{\gamma} = 0.90$ usually cannot be practically obtained with turbulent flow.

Table 1. Concentrations and temperatures: $\lambda = 0.01$, $Le = 200$

Re	Pr	ξ	$\bar{\gamma}_N$	$\bar{\gamma}$	$\bar{\theta}_N$	$\bar{\theta}$	$(\theta_1)_N$	θ_1
5000	1	0.51	0.198	0.190	0.118	0.154	0.200	0.238
		1.23	0.400	0.383	0.090	0.123	0.152	0.181
		2.24	0.601	0.578	0.060	0.083	0.100	0.124
		3.64	0.801	0.780	0.030	0.043	0.050	0.064
		5.69	0.901	0.887	0.015	0.022	0.025	0.033
5000	10	1.73	0.200	0.193	0.136	0.178	0.217	0.255
		4.18	0.404	0.387	0.107	0.143	0.166	0.200
		7.50	0.600	0.577	0.072	0.100	0.112	0.140
		13.3	0.800	0.780	0.036	0.052	0.066	0.072
		19.1	0.900	0.884	0.018	0.027	0.028	0.038
10^4	1	0.235	0.221	0.212	0.163	0.200	0.252	0.290
		0.523	0.409	0.390	0.130	0.167	0.198	0.232
		1.063	0.648	0.622	0.078	0.104	0.118	0.144
		1.747	0.817	0.794	0.040	0.057	0.062	0.079
		2.791	0.933	0.919	0.015	0.022	0.023	0.030
10^4	10	0.20	0.070	0.070	0.109	0.120	0.206	0.236
		0.70	0.203	0.202	0.178	0.200	0.256	0.295
		1.0	0.267	0.259	0.175	0.198	0.256	0.286
		2.0	0.450	0.432	0.137	0.158	0.137	0.226
		4.0	0.690	0.664	0.077	0.094	0.107	0.133
10^5	1	0.0111	0.218	0.207	0.318	0.325	0.414	0.444
		0.0291	0.394	0.380	0.322	0.350	0.388	0.419
		0.0651	0.614	0.590	0.210	0.239	0.252	0.283
		0.1155	0.794	0.770	0.112	0.134	0.135	0.158
		0.1830	0.912	0.839	0.048	0.062	0.058	0.074
10^5	10	0.01	0.084	0.084	0.153	0.187	0.250	0.277
		0.02	0.149	0.147	0.240	0.285	0.314	0.358
		0.04	0.249	0.242	0.308	0.354	0.365	0.408
		0.10	0.448	0.429	0.273	0.310	0.311	0.347
		0.40	0.865	0.844	0.067	0.085	0.077	0.095

Table 2. Nusselt and Sherwood numbers: $\lambda = 0.01$, $Le = 200$

Re	5000		10^4		10^5	
Pr	1	10	1	10	1	10
ξ_1	0.221	0.510	0.127	0.233	0.022	0.047
Nu_1	8.00	24.1	17.0	60.0	236	1430
Nu_{II}	8.43	23.8	21.1	60.7	484	1542
$\xi_1 \times 10^3$	0.24	0.29	5.2	8.0	0.3	0.3
Sh_1	1.08	333	288	876	7213	22054
Sh_I	102	320	264	825	6843	21515
ξ_1	0.185	0.440	0.091	0.164	0.011	0.018
Nu_0	5.75	14.54	9.62	25.6	58.7	239
Nu_{0A}	5.68	14.00	9.22	24.1	52.4	197
Nu_{0B}	5.23	11.30	9.11	19.6	57.5	124
ξ_2 for $\bar{\gamma} = 0.9$	5.7	19.4	2.4	8.0	0.173	0.461
x_2, (m)	6.52	22.2	8.0	26.6	21.6	57.5

The results in Table 1 for $Pr = 10$ and $Re = 10^4$ are essentially the same as those given for the same parameters in ref. [2]. This is because the interface diffusivity distribution is practically the same as that in ref. [2]. This region so dominates the problem that the higher diffusivity that exists in the remainder of the region does not influence the results significantly. For $Pr = 10$ and $Re = 10^5$, the values of $\bar{\gamma}_N$ in Table 1 are less than those of ref. [2] and correspond closely only for $\xi \geqslant 0.10$.

THE ALGEBRAIC SOLUTION

The primary purpose of the numerical calculations was to check the effectiveness of the algebraic solutions of ref. [1] when evaluated for the asymptotic values of the transfer numbers obtained from the numerical solution, as given in Table 2. Table 1 gives the results, $\bar{\gamma}$, $\bar{\theta}$ and θ_1, obtained from the algebraic solution. All correspond well with the results from the numerical solutions, though $\bar{\gamma} < \bar{\gamma}_N$ and $\bar{\theta} > \theta_N$. The mean concentration, $\bar{y}$, increases monotonically with ξ and, after the attainment of a maximum at relatively small values of ξ, the temperature, $\bar{\theta}$, decreases monotonically with ξ. The difference in the results from the numerical and from the algebraic solutions is consistent qualitatively with the fact that the average interface transfer numbers exceed the asymptotic values that are used for the algebraic solutions. However, the small differences between the numerical and algebraic results do indicate the latter to be satisfactory if the transfer numbers at the interface and at the wall can be specified correctly.

THE TRANSFER NUMBERS

For high Schmidt numbers, the interface transfer numbers are, as indicated in ref. [1], best obtained from correlations based on mass transfer experiments. Results like this have been used to provide diffusivity distributions for the interface like

$$\frac{\varepsilon}{\nu} = E\left(\frac{y'}{\Delta}\right)^2. \tag{1}$$

The assumption that the mixed mean concentrations (or temperatures) exist at the location where the diffusivity given by equation (1) equals $\varepsilon_{\rm m}/\nu$, a value typical of the average in the core of the turbulent flow, gives the interface transfer number, illustrated here by the Nusselt number, as

$$\frac{h\Delta}{K} = \frac{\sqrt{(E\nu/\alpha)}}{\tan^{-1}\left(\sqrt{\left(\frac{\varepsilon_{\rm m}}{\nu}\frac{\nu}{\alpha}\right)}\right)}. \tag{2}$$

In ref. [1] it was recommended that $\varepsilon_{\rm m}/\nu = a\Delta^+$, with $a = 0.057$. The higher eddy diffusivity given by the Van Driest model used in the present numerical calculations gives average values approximated better by $a = 0.14$. Using this with the value of E for the interface

$$E = 1.66\times10^{-7}Re^{1.678}(\Delta^+)^{4/3}$$

produces the interface transfer numbers, $Nu_{\rm II}$ and $Sh_{\rm I}$, that are contained in Table 2. For a Prandtl number of 10, these correspond well with the numbers, $Nu_{\rm I}$ and $Sh_{\rm I}$, given by the numerical solution. For a Prandtl number of 1, for which the Schmidt number is 200, the values of $Sh_{\rm I}$ and $Sh_{\rm I}$ correspond well, but at the highest Reynolds number the Nusselt number, $Nu_{\rm II}$, obtained from equation (2) exceeds very substantially the Nusselt number, $Nu_{\rm I}$, obtained from the numerical calculation. From the Van Driest model, the eddy diffusivity diminishes to zero at the free surface of the film, and the interface region is that where the diffusivity given by equation (1) is less than that given by the Van Driest model. Consequently, the extent of the interface region diminishes rapidly as the Reynolds number increases, and at $Re = 10^5$ the eddy diffusivity at the inner edge of the interface region is only 0.10 times the average diffusivity for the core flow. It is this deficiency that produces the poor correspondence between $Nu_{\rm I}$ and $Nu_{\rm II}$ for a Prandtl number of 1; much better results would have been obtained if Model 3 of ref. [4] had been adopted for the calculation. However, it must be noted that there are no experimental results available for the value of $Nu_{\rm I}$, as would be obtained by the heating of a liquid flowing down an adiabatic wall by its vapor.

For the Nusselt number at the wall, ref. [1] recommends the Colburn relation and values so obtained, $Nu_{\rm 0B}$, are contained in Table 1 with Nu_0, the Nusselt number given by the numerical solution. $Nu_{\rm 0B}$ is fairly acceptable for $Pr = 1$, but less so for $Pr = 10$ for which it is far too low for $Re = 10^5$. $Nu_{\rm 0A}$ is the value given by the Petukhor correlation; which gives a better comparison with Nu_0. In fact, it is expected that a completely appropriate turbulence model would produce values of Nu_0 close to those of $Nu_{\rm 0B}$, but still possibly different, since the values $Nu_{\rm 0B}$ are really for heating of the fluid by the wall, without any transfer at the free surface of the film.

CONCLUSIONS

The numerical results given here reinforce the conclusions of ref. [1] that the algebraic solution of the heat and mass transfer problem gives adequate results when the transfer numbers are used that are the asymptotic values for transfer at the interface and at the wall, obtained on the assumptions of relative independence of the transfers at the surface and at the wall.

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